

Guidelines on the Uncertainty of Force Measurements

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Version 3.0 (02/2022)



**Mass and Related
Quantities**

Authorship and Imprint

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The most important change to the previous version is the recategorisation of force machines based on their traceability path to the SI unit of force. Machines that derive their traceability via in situ calibrations using force transfer standards are termed Force Standard Machines (FSMs) while those which derive their traceability by any other means are termed SI-FSMs, with the calibration methodology for the two types of machine being described in detail. This replaces the Method A and Method B calibration approaches given in the previous version. Further changes were incorporated to make the document more useful to the reader. These include: illustrations of different types of force standard machine, an example uncertainty budget for deadweight force standard machines, additional information on a range of uncertainty contributions, an extended list of references, and full worked examples for ISO 376 and ISO 7500-1 calibrations.

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Guidelines on the Uncertainty of Force Measurements

Purpose

This document has been produced to enhance the equivalence and mutual recognition of calibration results obtained by laboratories performing calibrations in the field of force.

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1 INTRODUCTION

In a wide range of industrial applications, there is the need to measure a tensile or compressive force. These applications range from materials testing to industrial weighing, and from engine thrust measurement to the proof loading of bridge bearings. In each application, there will be an uncertainty requirement on the force measurement – the equipment used to make the measurement must be traceable to a realisation of the SI unit of force (the newton) within this required uncertainty.

The situation may vary slightly internationally, but this document is based on a country having one national metrology institute (NMI), or designated institute (DI), realising the newton in a number of force standard machines (FSMs) referred to as national force standard machines (NFSMs), and a number of calibration laboratories, generally accredited by their national accreditation body (NAB), using other FSMs to calibrate force-measuring instruments. These force-measuring instruments may then be used either to make direct industrial force measurements or to calibrate industrial force generating equipment, such as tensile testing machines.

The forces generated within calibration laboratory FSMs are usually traceable to those generated within NFSMs (or other FSMs of the required uncertainty in generated force) through exercises using force transfer standards, and the accredited calibration and measurement capability (CMC) of the calibration laboratory will be based on the results of these exercises.

NOTE: The NAB is likely to be the organisation responsible for validating these CMCs and requesting information from the calibration laboratory regarding the traceability of their FSMs. Many NABs base their force calibration laboratory accreditation procedures on the recommendations given within this document.

Alternatively, instead of being calibrated by force transfer standards, calibration laboratory FSMs may derive their traceability to the newton via a mathematical model of the force generation system and SI traceable measurements of all relevant measurands, in which case they are defined to be SI reference FSMs (SI-FSMs). For such machines, comparisons with NFSMs to validate the claimed CMC may be required by the NAB. To clarify, all machines that derive their traceability by methods other than *in situ* calibration by force transfer standards are classified as SI FSMs.

Most, but not all, NFSMs are SI-FSMs, the others being machines in which the generated force is determined by calibrations using force transfer standards. NFSMs have their CMCs validated by international comparisons with other NMIs/DIs.

NOTE: It should be noted that the distinction between SI-FSMs and other FSMs is not the same as that between primary measurement standards and secondary measurement standards, as defined by the VIM [1]. These other FSMs will always be secondary measurement standards as their traceability to the SI is via measurements (made by force transfer standards) in the unit of force. SI-FSMs may be either secondary or primary measurement standards, depending on whether their traceability to the SI is via measurements of force or via measurements of other quantities.

The diagram given in Figure 1 summarises the traceability situation (the items in the dashed box can be repeated more than once in the chain from SI-FSM to Force-measuring Instrument).

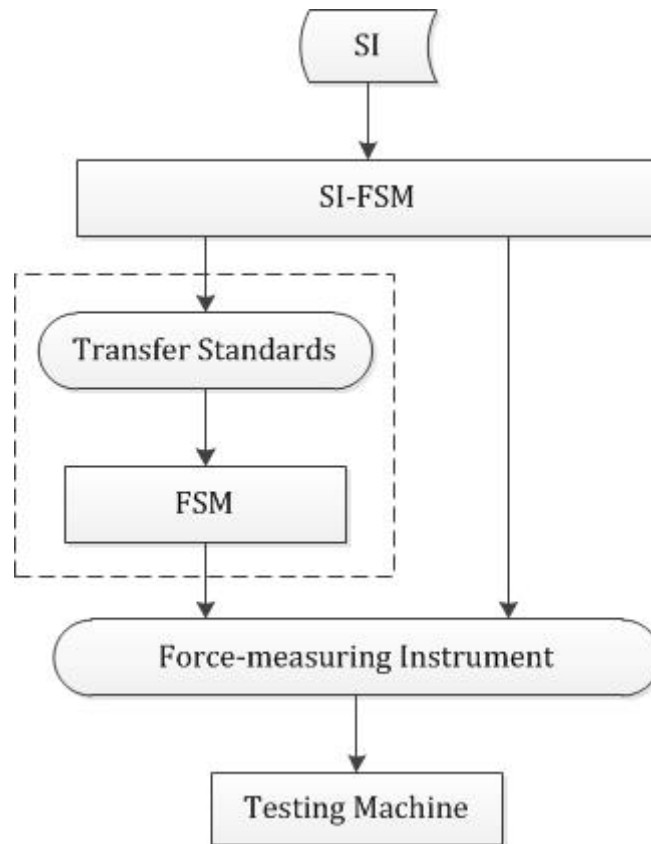


Figure 1. Force traceability of Testing Machines via SI-FSMs to the realisation of the SI unit.

The calibration of a force-measuring instrument in an FSM will generally be carried out in accordance with a documented procedure, such as ISO 376 [2], and the uncertainty of the calibration results will be dependent on the machine's CMC, as well as on the performance of the instrument during the calibration. ISO 376 classification criteria include the magnitude of the uncertainty of the applied force and, for high quality force transducers, this value is often the dominant uncertainty contribution. For this reason, the CMC of an FSM is often set to be equal to the uncertainty of the applied force, ignoring any other uncertainty contributions that may be present during the calibration.

Similarly, the uncertainty of the calibration of the industrial force-generating equipment will be partly dependent on the uncertainty arising from the force-measuring instrument, and the uncertainty of any subsequent force measurements will depend in part on the uncertainty associated with the force-generating equipment.

It can be seen that the uncertainty of the final force measurement is dependent on all of the previous traceability stages, and this document aims to give guidance on how to estimate all of these contributions.

The above traceability situation strictly covers only static force measurement, whereas a significant number of industrial force measurement applications, such as fatigue and impact testing, are dynamic in nature – additional uncertainty considerations need to be made when dealing with such measurement areas.

2 SCOPE

The scope of this document is to give guidance on the estimation of force measurement uncertainty in a range of different areas, namely:

- uncertainty of forces generated by SI reference force standard machines
- uncertainty of forces generated by other force standard machines
- uncertainty of forces measured by force-measuring instruments
- uncertainty of forces generated by industrial force-generating equipment

In each of these cases, the uncertainty determination is based on two major components – the uncertainty obtained during the calibration of the equipment and the uncertainty resulting from the equipment's subsequent use.

In addition, other uncertainty contributions that may need to be considered when dealing with dynamic force measurement applications are briefly discussed.

3 SYMBOLS AND ABBREVIATIONS

Symbol	Description	Unit*
a_{drift}	half-width of relative variation due to drift	-
b'	ISO 376 relative repeatability error	%
c	relative creep error	%
CMC	calibration and measurement capability	-
d	degree of equation	-
DI	designated institute	-
f_0	ISO 376 relative zero error	%
F	force	N
F_{cal}	calibration force	N
$F_{\text{si-fsm}}$	force generated by SI-reference force standard machine	N
F_{min}	minimum calibration force	N
F_{nom}	nominal force	N
FSM	force standard machine	-
g	acceleration due to gravity	$\text{m}\cdot\text{s}^{-2}$
i_{30}	output 30 s after application or removal of maximum calibration force	$\text{mV}\cdot\text{V}^{-1}$
i_{300}	output 300 s after application or removal of maximum calibration force	$\text{mV}\cdot\text{V}^{-1}$
i_f	final indicator reading – i.e. after force application	$\text{mV}\cdot\text{V}^{-1}$
i_0	original indicator reading – i.e. before force application	$\text{mV}\cdot\text{V}^{-1}$
ISO	International Organization for Standardization	-
k	coverage factor	-

K	force instrument's temperature coefficient	K^{-1}
K_{ts}	calibration coefficient of transfer standard	$N \cdot (mV \cdot V^{-1})^{-1}$
m	mass	kg
M_b	bending moment	N·m
M_{b_ref}	reference bending moment	N·m
NAB	national accreditation body	-
NFSM	national force standard machine	-
NMI	national metrology institute	-
r	resolution	N
SI-FSM	SI-reference force standard machine	-
TC_S	temperature coefficient of sensitivity per 10 K	%
TC_0	temperature coefficient of zero signal per 10 K	%
$u(F)$	standard uncertainty of force	N
$u(g)$	standard uncertainty of acceleration due to gravity	$m \cdot s^{-2}$
$u(m)$	standard uncertainty of mass	kg
$u(\rho_a)$	standard uncertainty of air density	$kg \cdot m^{-3}$
$u(\rho_m)$	standard uncertainty of density of weight	$kg \cdot m^{-3}$
w_{approx}	relative standard uncertainty due to approximation to interpolation equation	-
w_c	combined relative standard uncertainty	-
w_{cal}	relative standard uncertainty due to calibration of transfer standard	-
w_{corr}	relative standard uncertainty associated with correction value	-
$w(D)$	relative standard uncertainty due to drift	-
$w(d_{fsm})$	relative standard uncertainty associated with force generation in force standard machine	-
w_{drift}	relative standard uncertainty due to drift of transfer standard	-
$w(F_{si-fsm})$	relative standard uncertainty of force generated by SI-reference force standard machine	-
w_i	relative standard uncertainty associated with parameter i	-
$w(K_{ts})$	relative standard uncertainty of force value indicated by transfer standard	-
w_{M_b}	relative standard uncertainty of bending moment	-

$W_{\text{ref_instab}}$	relative standard uncertainty of reference force transducer's long-term instability	-
$W_{\text{ref_tra}}$	relative standard uncertainty of calibration of reference force transducer	-
W_{rep}	relative standard uncertainty due to repeatability	-
W_{res}	relative standard uncertainty due to resolution	-
W_{rev}	relative standard uncertainty due to reversibility	-
W_{rv}	relative standard uncertainty of reference value	-
W_{std}	relative standard uncertainty due to transfer standard	-
W_{TC_S}	relative standard uncertainty of temperature sensitivity	-
W_{TC_0}	relative standard uncertainty of zero signal temperature coefficient	-
W_{temp}	relative standard uncertainty due to temperature effects	-
$w(X)$	relative standard uncertainty of mean deflection	-
W	relative expanded uncertainty	-
W_{cal}	relative expanded uncertainty of instrument calibration	-
W_{CMC}	relative expanded uncertainty of force generated by force calibration machine, equivalent to CMC	-
$W_{\text{si-fsm}}$	relative expanded uncertainty of force generated by SI-reference force standard machine	-
$W_{\text{ref_instab}}$	relative expanded uncertainty of reference force transducer's long-term instability	-
$W_{\text{ref_tra}}$	relative expanded uncertainty of calibration of reference force transducer	-
W_{rv}	relative expanded uncertainty of reference value	-
W_{ts}	relative expanded uncertainty of force value indicated by transfer standard	-
X	mean deflection	$\text{mV}\cdot\text{V}^{-1}$
X_{fsm}	mean deflection in force standard machine	$\text{mV}\cdot\text{V}^{-1}$
X_{fsm_i}	individual deflection in force standard machine	$\text{mV}\cdot\text{V}^{-1}$
X_i	individual deflection value in run i	$\text{mV}\cdot\text{V}^{-1}$
X_N	deflection at maximum calibration force	$\text{mV}\cdot\text{V}^{-1}$
\bar{X}_r	mean deflection from ISO 376 runs 1, 3, and 5	$\text{mV}\cdot\text{V}^{-1}$
δ_r	sum of squared deviations between mean deflection and calculated value	$(\text{mV}\cdot\text{V}^{-1})^2$
Δd_d	relative deviation for decremental forces	-

Δd_i	relative deviation for incremental forces	-
Δd_{\max}	absolute value of maximum relative deviation between reference value and value obtained in force calibration machine	-
ΔM_b	data sheet specification of the influence of bending moment	%
ΔT	range of temperature during calibration	K
ΔT_0	temperature difference to last tare	K
ΔT_{cal}	temperature difference to instrument calibration	K
ρ_a	density of air	$\text{kg}\cdot\text{m}^{-3}$
ρ_m	density of weight	$\text{kg}\cdot\text{m}^{-3}$

* In the table it is assumed that force transducers and reference standards are based on resistive strain gauge technology, with outputs and deflections measured in $\text{mV}\cdot\text{V}^{-1}$. For other types of transducer and instrumentation, this measurement unit may be different, e.g. pC, V, mA, mm, Hz.

4 FORCE STANDARD MACHINES (FSMs)

This section introduces the most common types of FSM, namely:

- deadweight
- hydraulic amplification
- lever amplification
- reference (single or multiple) transducer system

All FSMs can have their traceability derived from calibrations performed using force transfer standards which have themselves been calibrated in an FSM of lower uncertainty. The methodology for performing such calibrations and estimating the resultant CMC is given in Section 5.

However, for machines of the types described within this section, it is also possible to derive traceability for the estimated value of generated force by other methods, thus treating them as SI-FSMs. For the various types of FSM considered in the following, the major factors which may influence the uncertainty of such estimations are given.

For SI-FSMs, it may be necessary to perform comparisons with NFSMs to validate their performance, and the procedure given in Section 5 can be tailored to suit this purpose. Further details are given in Section 4.5.

4.1 Deadweight force standard machines

The net downward vertical force F (in N) generated by a weight of mass m (in kg) and density ρ_m (in $\text{kg}\cdot\text{m}^{-3}$) suspended in air of density ρ_a (in $\text{kg}\cdot\text{m}^{-3}$) in the Earth's gravitational field of strength g (in $\text{m}\cdot\text{s}^{-2}$) is given by:

$$F = mg \left(1 - \frac{\rho_a}{\rho_m} \right) \quad (1)$$

The uncertainties in the four variables on the right-hand side of this equation can be combined to determine the uncertainty in the calculated value of force (where $u(x)$ is the standard uncertainty associated with variable x):

$$\left(\frac{u(F)}{F}\right)^2 \approx \left(\frac{u(m)}{m}\right)^2 + \left(\frac{u(g)}{g}\right)^2 + \left(\frac{u(\rho_a)}{\rho_m}\right)^2 + \frac{\rho_a^2}{\rho_m^2} \left(\frac{u(\rho_m)}{\rho_m}\right)^2 \quad (2)$$

NOTE: This formula is not strictly correct, as it assumes that the variables are independent, ignoring any correlation between the mass and the density of the weight, resulting from its calibration [3]. A more correct formulation is:

$$\left(\frac{u(F)}{F}\right)^2 \approx \left(\frac{u(m)}{m}\right)^2 + \left(\frac{u(g)}{g}\right)^2 + \left(\frac{u(\rho_a)}{\rho_m}\right)^2 - \frac{\rho_a(2\rho_{\text{cal}} - \rho_a)}{\rho_m^2} \left(\frac{u(\rho_m)}{\rho_m}\right)^2$$

where ρ_{cal} is the air density at the time of the calibration of the mass – when using this equation, it is important that the values of the weight's density and its associated uncertainty are the same as those used during its calibration. If ρ_{cal} is not known, realistic values for it and ρ_a should be selected to maximise the contribution of the uncertainty of the density of the weight to the uncertainty of the force – see [4] for further information on air density considerations. Equations in terms of volume rather than density can also be found in [3].

The uncertainty associated with each of the variables should take into account its variation over time – air density and gravitational acceleration will vary throughout any given day, whereas the mass value is likely to be subject to longer-term drift, caused by wear, contamination, and surface stability.

In the case where the true mass value of the weight is not known, but its conventional mass value m_c is (i.e. m_c is the mass of a weight of density $8\,000\text{ kg}\cdot\text{m}^{-3}$ which will balance the unknown weight in air of density $1.2\text{ kg}\cdot\text{m}^{-3}$) – the conventional mass is normally the value given on a mass calibration certificate – (1) and (2) are amended as follows:

$$F = m_c g \left(1 - \frac{1.2}{8\,000} + \frac{(1.2 - \rho_a)}{\rho_m}\right) \quad (3)$$

and

$$\left(\frac{u(F)}{F}\right)^2 \approx \left(\frac{u(m_c)}{m_c}\right)^2 + \left(\frac{u(g)}{g}\right)^2 + \left(\frac{u(\rho_a)}{\rho_m}\right)^2 + \left(\frac{1.2 - \rho_a}{\rho_m}\right)^2 \left(\frac{u(\rho_m)}{\rho_m}\right)^2 \quad (4)$$

NOTE: As with (2), (4) is not strictly correct for the same reasons, with a more accurate version being:

$$\left(\frac{u(F)}{F}\right)^2 \approx \left(\frac{u(m_c)}{m_c}\right)^2 + \left(\frac{u(g)}{g}\right)^2 + \left(\frac{u(\rho_a)}{\rho_m}\right)^2 + \frac{[(\rho_a - \rho_{\text{cal}})^2 - (\rho_{\text{cal}} - 1.2)^2]}{\rho_m^2} \left(\frac{u(\rho_m)}{\rho_m}\right)^2$$

with the same criteria applying as specified in the previous note.

The uncertainty budget for the machine also needs to consider possible force-generating mechanisms other than gravity and air buoyancy, including magnetic, electrostatic, and aerodynamic effects.

For machines in which the applied force is not a pure deadweight – where, for example, the weight of the loading frame is tared off with a lever and counterweight, or the

scalepan is stabilised with a guidance system – the effect of any frictional or unbalanced forces needs to be additionally incorporated within the uncertainty budget, at each force within the machine’s range.

The ability of the machine to hold the force transducer at the correct alignment – i.e. with its measuring axis vertical and concentric to the applied force – at each applied force will have an effect on the magnitude of the force vector applied to the transducer’s measuring axis, and this should also be included in the uncertainty budget. Other machine-specific characteristics, such as compression platen stiffness and side force generation, may also affect transducer output (this will depend on the transducer’s sensitivity to such effects) but do not contribute to the uncertainty of the applied force along the transducer’s measuring axis – and this is the uncertainty to which a CMC value refers. An example uncertainty budget is given in the following table, in which the sensitivity coefficient for each uncertainty contribution $u(x)$ is calculated from the corrected version of (4) as $u(F)/u(x)$, assuming all other uncertainty contributions to be zero.

MODEL:

$$F = m_c g \left(1 - \frac{1.2}{8\,000} + \frac{(1.2 - \rho_a)}{\rho_m} \right)$$

$$\left(\frac{u(F)}{F} \right)^2 \approx \left(\frac{u(m_c)}{m_c} \right)^2 + \left(\frac{u(g)}{g} \right)^2 + \left(\frac{u(\rho_a)}{\rho_m} \right)^2 + \frac{[(\rho_a - \rho_{cal})^2 - (\rho_{cal} - 1.2)^2]}{\rho_m^2}$$

Uncertainty Component				Uncertainty Contribution				Sensitivity Coefficient		Unc
Definition	Symbol	Value	Unit	Value	Unit	Distribution	Divisor	Value	Unit	
Conventional mass	m_c	1 019.332	kg	0.003	kg	normal	2	9.81	N·kg ⁻¹	
Acceleration due to gravity	g	9.811 819	m·s ⁻²	0.000 002	m·s ⁻²	normal	2	1 019	N·(m·s ⁻²) ⁻¹	
Density of weight	ρ_m	7 907	kg·m ⁻³	50	kg·m ⁻³	normal	2	0.000 011	N·(kg·m ⁻³) ⁻¹	
Density of air	ρ_a	1.200	kg·m ⁻³	0.024	kg·m ⁻³	normal	2	1.26	N·(kg·m ⁻³) ⁻¹	
sum of variances										
standard uncertainty										
expanded uncertainty (k = 2)										
Force	F	10 000.000	N							
Density of air ρ_a to maximise $u(\rho_m)$ sensitivity	ρ_a	1.240	kg·m ⁻³							
Density of air during mass calibration	ρ_{cal}	1.160	kg·m ⁻³							

Figure 2 shows a typical deadweight force standard machine.

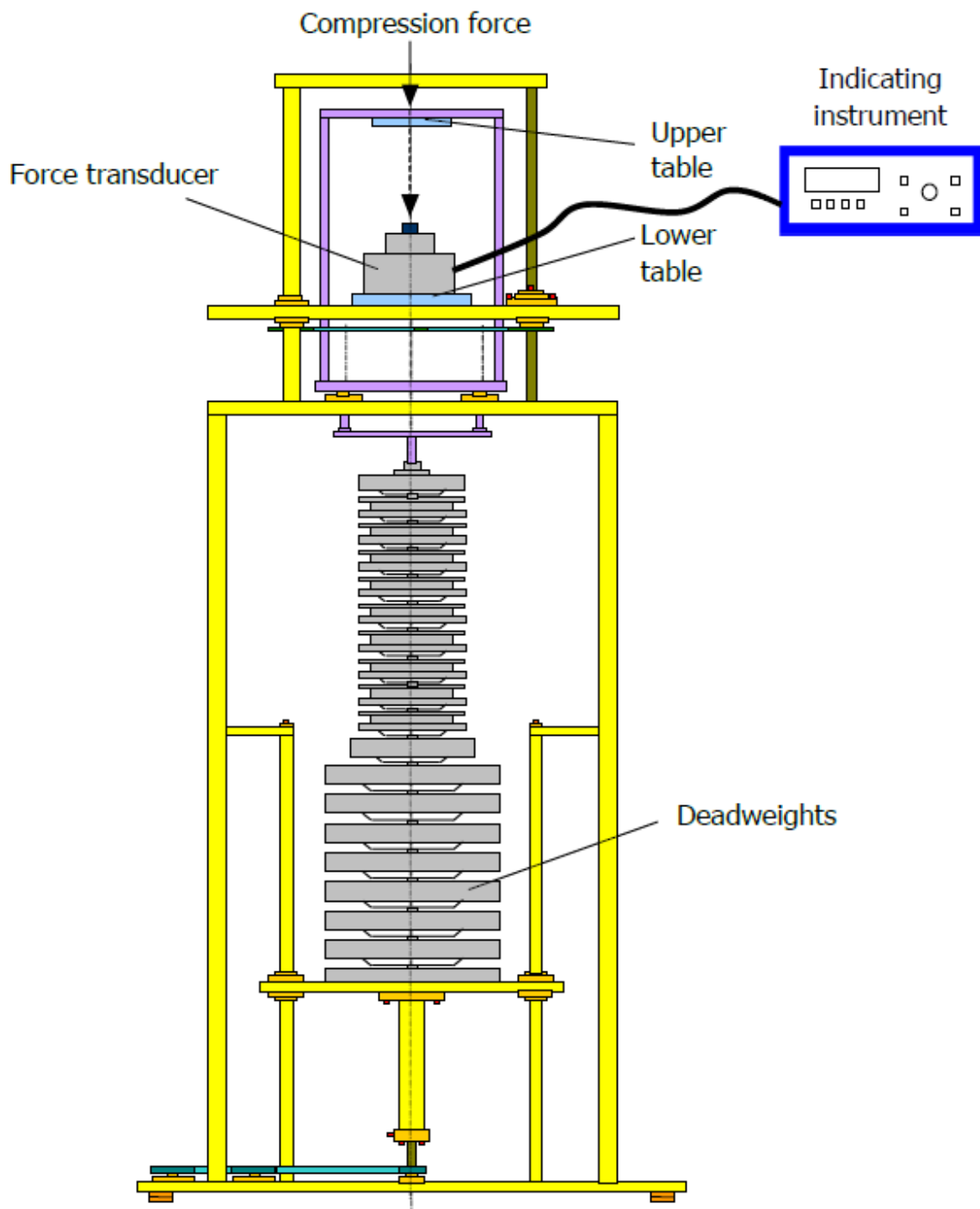


Figure 2. Typical deadweight force standard machine

4.2 Hydraulic amplification force standard machines

In a hydraulic amplification machine, a deadweight force is amplified by the use of a hydraulic system with piston/cylinder assemblies of different effective areas, increasing

the force by a factor approximately equal to the ratio of the two areas. Where the traceability of this larger force is directly derived from this amplification model, the uncertainty contributions that need to be considered will include, but are not limited to, the following:

- uncertainty of the deadweight force (see 4.1 for details)
- uncertainty of both piston/cylinder assembly dimensional measurements or uncertainty of determining the amplification ratio
- uncertainty due to pressure differences throughout the hydraulic circuitry, caused by hydraulic fluid flow and vertical height
- uncertainty due to effect of temperature on area ratio (thermal expansion, at possibly different rates, of piston/cylinder assemblies) and pressure drops (temperature dependence of hydraulic fluid's viscosity)
- uncertainty due to effect of pressure on area ratio (elastic distortion of piston/cylinder assemblies)
- uncertainty due to instability of control system
- uncertainty due to friction/hysteresis within piston/cylinder assemblies or mechanical guidance systems
- uncertainty associated with setting the initial zero force point

Where possible, corrections should be made for the estimated effect of any of these components on the magnitude of the generated force. The standard uncertainties associated with these corrections, together with the standard uncertainties due to any effects that cannot be corrected for, should be combined in quadrature (if it can be demonstrated that the effects are not correlated) and then multiplied by a coverage factor to derive an expanded uncertainty for the generated force.

Figure 3 shows a typical hydraulic amplification force standard machine.

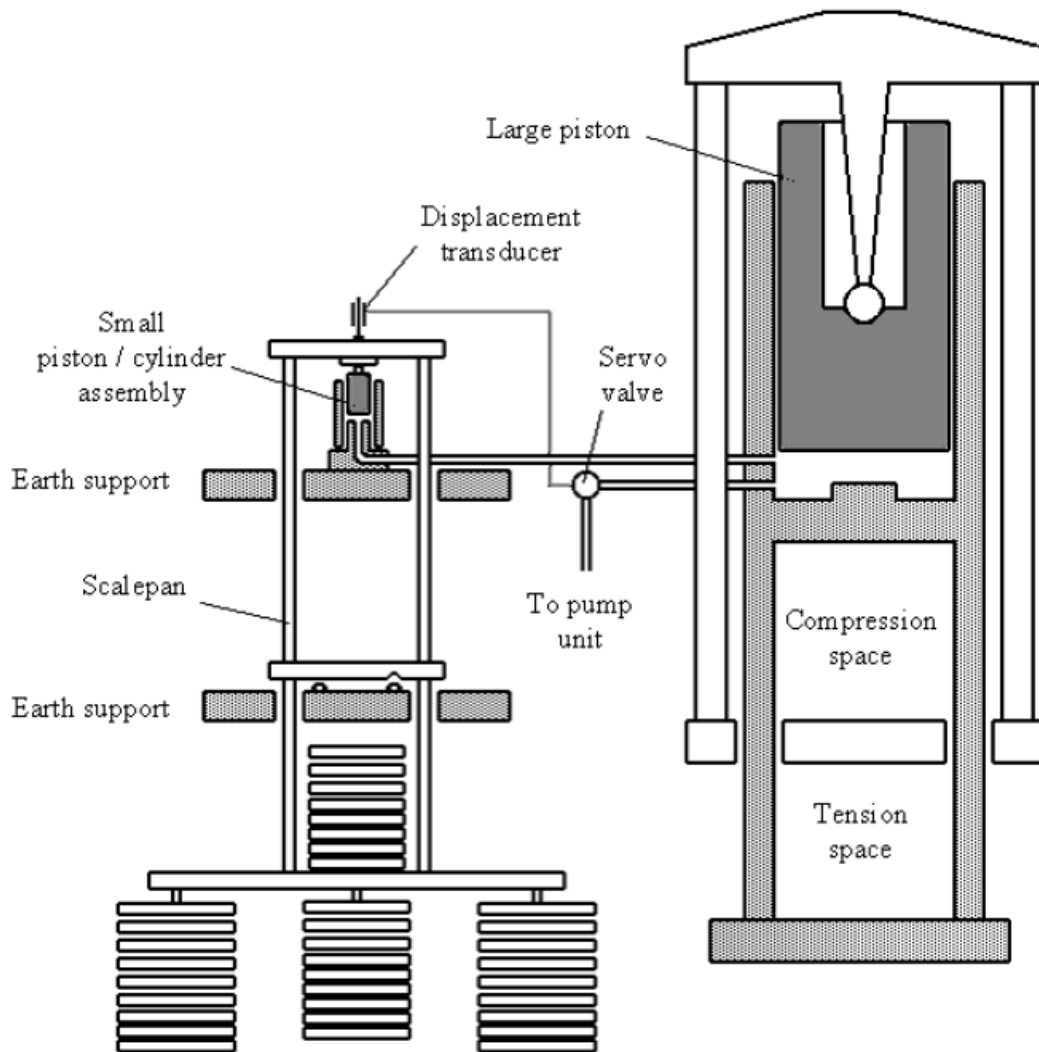


Figure 3. Typical hydraulic amplification force standard machine

4.3 Lever amplification force standard machines

In a lever amplification machine, a deadweight force is amplified by the use of one or more mechanical lever systems, increasing the force by a factor approximately equal to the ratio of the lever arm lengths. Where the traceability of this larger force is directly derived from this amplification model, the uncertainty contributions that need to be considered will include, but are not limited to, the following:

- uncertainty of the deadweight force (see 4.1 for details)
- uncertainty of the lever system dimensional measurements or uncertainty of determining the lever ratio
- uncertainty due to friction within the lever systems
- uncertainty due to effect of temperature on lever arm ratio (thermal expansion, at possibly different rates, of lever systems)

- uncertainty due to effect of applied force magnitude on lever arm ratio (elastic distortion of lever systems)
- uncertainty due to instability of control system
- uncertainty due to alignment of generated force with transducer's measuring axis
- uncertainty due to positional reproducibility of moveable parts
- uncertainty due to wear/stability of knife-edges, if used

Where possible, corrections should be made for the estimated effect of any of these components on the magnitude of the generated force. The standard uncertainties associated with these corrections, together with the standard uncertainties due to any effects that cannot be corrected for, should be combined in quadrature (if it can be demonstrated that the effects are not correlated) and then multiplied by a coverage factor to derive an expanded uncertainty for the generated force.

Machines incorporating a jockey weight on a lever can be treated in a similar manner.

Figure 4 shows a typical lever amplification force standard machine.

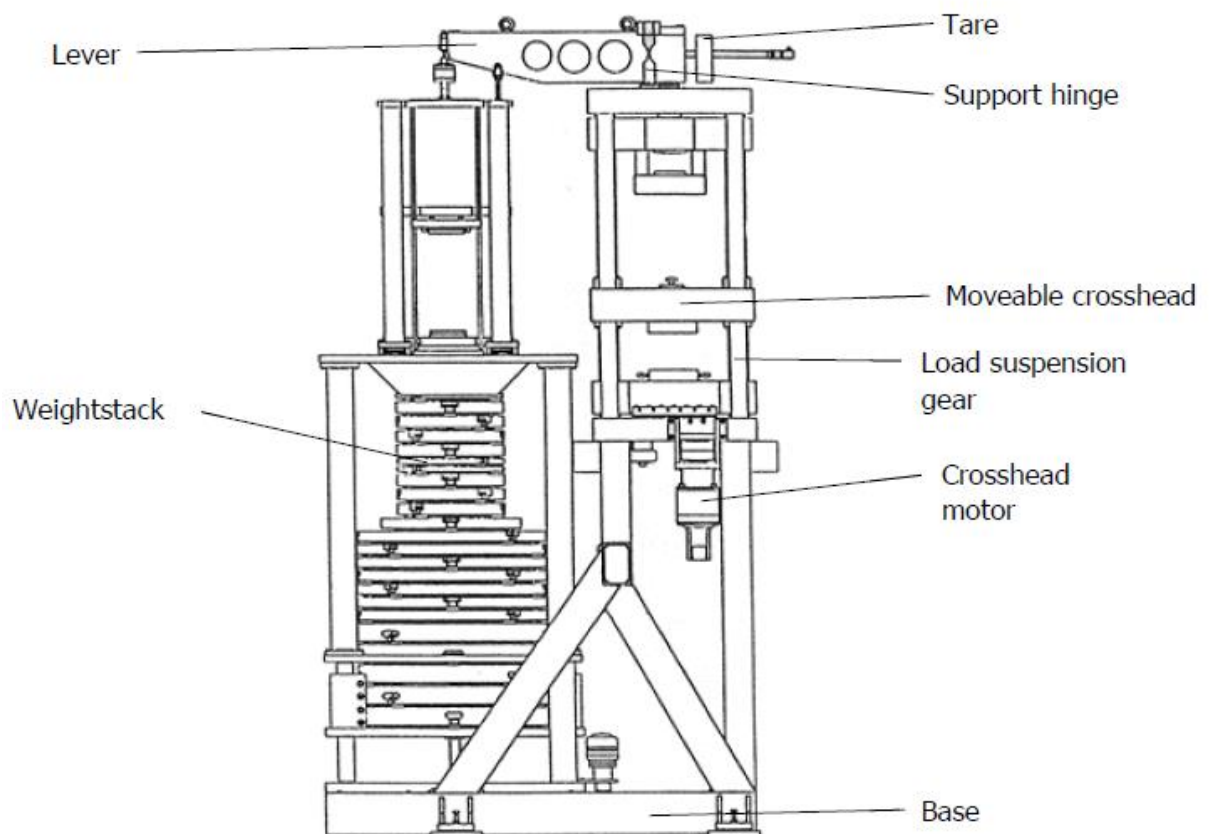


Figure 4. Typical lever amplification force standard machine

4.4 Reference transducer system force standard machines

These machines are based on one or more reference force transducers that have previously been individually calibrated in another FSM. These reference transducers are then loaded (either singly, or in parallel as a build-up system) in series with the instrument being calibrated. The generated force is calculated as the sum of the forces being measured by the individual transducers (details of an EMRP-funded research project on force traceability within the meganewton range, which focused on the performance of build-up systems, can be found at <https://www.euramet.org/project-sib63>)

For this type of machine, the uncertainty contributions that need to be considered will include, but are not limited to, the following:

- uncertainty of the calibrations of the individual transducers (for guidance, see Section 6)
- uncertainty due to use of transducers after their calibration (for guidance, see Section 7.1)
- uncertainty due to alignment of transducers with the measuring axis of the transducer under calibration
- uncertainty due to stability/performance of control system and data acquisition methodology

Where possible, corrections should be made for the estimated effect of any of these components on the magnitude of the generated force. The standard uncertainties associated with these corrections, together with the standard uncertainties due to any effects that cannot be corrected for, should be combined in quadrature (if it can be demonstrated that the effects are not correlated) and then multiplied by a coverage factor to derive an expanded uncertainty for the generated force.

Figure 5 shows a typical reference transducer system FSM.

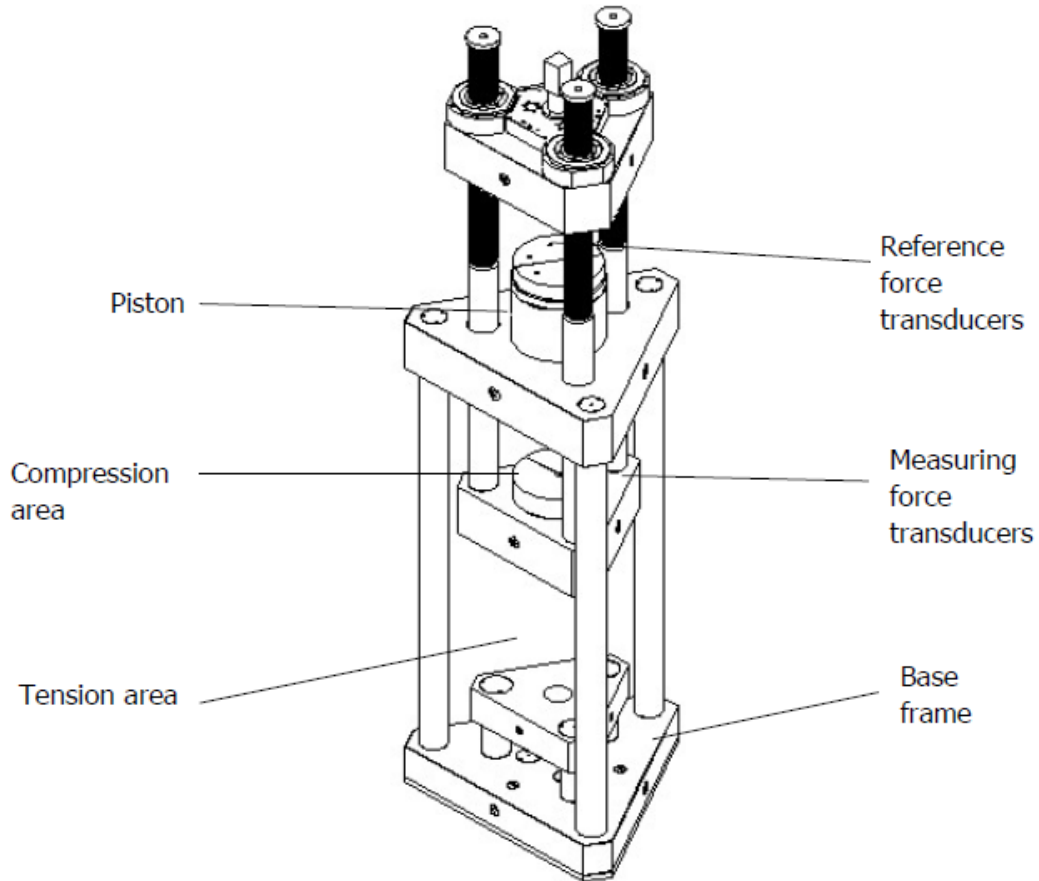


Figure 5. Typical reference transducer system force standard machine

4.5 Validation of SI-FSM CMCs

SI-FSMs (located both at NMIs/DIs and in calibration laboratories) are not calibrated as complete machines, rather their traceability is derived from measurements of parameters such as mass, gravity, lever length, piston area, and even force (in reference transducer machines), and the uncertainty associated with the generated force (and the laboratory's claimed CMC) is calculated from the uncertainties associated with these measurements, together with the other contributions detailed earlier in this section. It is necessary to perform comparisons between SI-FSMs and appropriate NFSMs using high quality transfer standards – the procedure for this work may be as described in Section 5 but the results need to be analysed in a different way, as they are comparison exercises rather than calibrations. The analysis needs to demonstrate whether the results from the two machines are metrologically compatible – one method for assessing this is described in [5] and involves determining whether or not the E_n values calculated across the range of applied force exceed unity. If these values do exceed unity, it is not sufficient simply to increase the CMC to reduce the E_n value to an acceptable level, but the whole uncertainty budget associated with the SI-FSM (and with the comparison procedure) should be reviewed to the satisfaction of the NAB.

When the procedure specified in Section 5 is used as the basis for the comparison of the SI-FSM with an appropriate NFSM, it is recommended that the expanded uncertainty of the transfer standard's reference value (determined in Step 3) is a factor

of at least two smaller than the SI-FSM's CMC, to enable variations between the machines to be clearly identified.

The uncertainty of measurement associated with the force scales realised at NMIs is ensured by means of international intercomparisons. The expanded relative uncertainty of measurement with which force values can be generated by deadweight force standard machines is stated by various NMIs as being as low as 1×10^{-5} . In practice, however, when different deadweight force standard machines are used to calibrate the same force transducer, the differences between the results may often be significantly greater, due to mechanical interaction effects. This became evident in BCR and WECC interlaboratory comparisons, based on force transducer calibrations carried out in 1987 and 1991 respectively [6, 7].

5 CALIBRATION OF FSMs

5.1 CMCs of calibration laboratory FSMs

The CMCs achieved by FSMs depend on the type of force generation - Table 5.1 shows typical values for the different FSM types in calibration laboratories. The uncertainty with which values of force are realised by deadweight FSMs may be calculated, as in Section 4.1, to be as small as 1×10^{-5} . However, if traceability to NFSMs is required or if the claimed CMC needs to be validated via a comparison with an NFSM, the demonstration of a CMC smaller than 5×10^{-5} may be either technically infeasible or simply too expensive. In most cases the requirements of the calibration laboratory are satisfied if a CMC of 1×10^{-4} can be achieved. This enables the calibration laboratory to calibrate force-measuring instruments to the best classification specified within ISO 376.

In hydraulic and lever amplification machines, the lower values for the CMC can only be achieved by the correction of any systematic component of the amplification effect. For the determination of the CMC of a comparator type FSM, the machine's incorporated reference force transducer(s) should, if possible, first be calibrated in a suitable FSM to determine relevant metrological characteristics.

Table 5.1: Typical CMCs of calibration laboratory FSMs

Type of machine	Typical range of CMCs (expanded relative uncertainty)
Deadweight	5×10^{-5} to 1×10^{-4}
Hydraulic amplification	1×10^{-4} to 5×10^{-4}
Lever amplification	1×10^{-4} to 5×10^{-4}
Comparator with one or multiple reference force transducers	2×10^{-4} to 5×10^{-3}

The CMC of an SI-FSM is determined from the machine's uncertainty budget, considering all significant influence quantities, as described in Section 4. This CMC is validated by comparison against an appropriate NFSM. For other FSMs, CMCs are derived from the results of calibration exercises using force transfer standards, and the methodology for this work is given within the following sections.

5.2 Determination of the machine's CMC

It should be borne in mind that the machine's CMC cannot be lower than the uncertainty of the reference value derived from the calibration of the transfer standards in the FSM

providing the traceability link, usually an SI-FSM (for clarity, the following sections assume that this machine is an SI-FSM). To minimise the CMC it is important to ensure that this reference value uncertainty is as low as possible.

Recommendations and good practice for reducing or minimising the uncertainty of the reference value include the following, some or all of which may need to be implemented to meet the requirements of the specific exercise:

- Use of an SI-FSM with a low CMC
- Use of transfer standards in the range above 40 % of their maximum capacity, to minimise interaction effects and reduce uncertainty due to resolution
- The use of high-quality instrumentation with a resolution (defined as one increment of the indicator's least significant count plus half the range of fluctuation at zero force) of better than 1 part in 200 000 at each calibration force – this is the assumption made in the following analysis in which no uncertainty contribution due to resolution is considered (if the magnitude of the resolution is significant with respect to the uncertainty of the applied force or the repeatability of the results, a resolution component should be included in the uncertainty calculations)
- Careful selection of instrumentation regarding characteristics such as non-linearity, temperature effects, and drift, so that these uncertainty contributions of the instrumentation can be neglected, and use of the same instrumentation for all measurements

To determine the machine's CMC, the following measurement plan should be applied:

- Selection of at least two force transducers as transfer standards to cover the whole force range of the FSM. At the common forces where the force ranges covered by different transducers meet, this force shall be measured by both transducers - separate transfer standards for tension and compression may also be needed.
- Calibration of these transfer standards in an SI-FSM. The measurements shall be carried out in at least three rotational positions and shall include hysteresis measurements – to determine repeatability, the measurements are to be repeated once in at least one of the rotational positions. The number and selection of force values used will be dependent on the type of FSM to be calibrated.
- Calibration of the transfer standards in the FSM. The measurement procedure will be similar to the calibration of the transfer standard in the SI-FSM, ideally replicating the time and loading profiles.
- Recalibration of the transfer standards in the SI-FSM to determine the overall reference values and the magnitude of any drift of the transfer standards throughout the exercise. Historical drift data based on repeated calibrations of the transfer standards over a longer period may also be used to inform this determination. It is also permitted to follow an FSM – SI-FSM – FSM calibration protocol, with the reference value taken from the SI-FSM calibration and the drift determined from the two FSM calibrations (this protocol is not recommended for comparator-type machines as any transfer standard drift cannot be separated from any drift of the FSM).
- For each transfer standard at each nominal force level, determination of the relative deviation between the reference value and the value obtained in the FSM.

The machine's CMC can now be determined following a five-step process

- Step 1 - Determination of the uncertainty of the force generated by the SI FSM

- Step 2 - Determination of the calibration uncertainty of the transfer standard in the SI FSM
- Step 3 - Determination of the uncertainty of the transfer standard's reference value
- Step 4 - Determination of the uncertainty of force generation in the FSM
- Step 5 - Determination of the FSM's CMC

Step 1 - Determination of the uncertainty of the force generated by the SI-FSM

The expanded relative uncertainty W_{fsm} with which the unit of force is realised by a typical SI-FSM is calculated following the guidance in Section 4 typical values are given in Table 5.2.

Step 2 - Determination of the calibration uncertainty of the transfer standard in the SI-FSM

The quantity determined in the calibration of a force transducer used as a transfer standard for the selected force steps is its calibration coefficient K_{ts} which is the ratio of the applied force $F_{\text{si-fsm}}$ to the deflection X indicated by the force transducer:

$$K_{\text{ts}} = \frac{F_{\text{si-fsm}}}{X} \quad (5)$$

To eliminate the influence of the rotation effect, the deflection X is the mean value of n rotational positions of the transducer uniformly spaced around its axis.

$$X = \frac{1}{n} \sum_{i=1}^n X_i \quad (6)$$

where X_i are the deflections indicated by the force transducer in the different rotational positions.

The relative variance of the mean deflection is given by:

$$w^2(X) = \frac{1}{n(n-1)} \times \sum_{i=1}^n \left(\frac{X_i - X}{X} \right)^2 \quad (7)$$

Alternatively, if the number of rotational positions is high enough ($n > 3$) and they are at equally distributed orientations, the relative variance of the mean deflection can be derived from the residuals of a sinusoidal fit of deflection against orientation.

The combined relative standard uncertainty of the value of force indicated by the transfer standard $w(K_{\text{ts}})$ and its relative expanded uncertainty W_{ts} can be determined by the following equations:

$$w(K_{\text{ts}}) = \sqrt{w^2(X) + w^2(F_{\text{si-fsm}})} \quad (8)$$

$$W_{\text{ts}} = k \times w(K_{\text{ts}}) \quad (9)$$

where k is the coverage factor required to give a confidence level of 95 % - this value will depend on the relative Type A and Type B uncertainty contributions, and can be calculated using the Welch-Satterthwaite equation [8].

Step 3 - Determination of the uncertainty of the transfer standard's reference value

As the transfer standard is used throughout a finite period of time, the influence of any sensitivity drift D has to be taken into account by incorporating a further relative uncertainty contribution as follows:

$$w^2(D) = \frac{a_{\text{drift}}^2}{3} \quad (10)$$

where its value is estimated by a rectangular probability distribution of half-width a_{drift} of relative variation of sensitivity. If it can be shown that the drift is time-dependent, the rectangular distribution may be replaced by a triangular one (using a divisor of 6 instead of 3). This replacement is only justified if the comparison measurements are made during a short period of time (typically about one month) and the calibration of the FSM is performed approximately mid-way between the two calibrations in the SI-FSM.

The expanded relative uncertainty of the reference value is evaluated as follows:

$$W_{\text{rv}} = k \times \sqrt{w^2(K_{\text{ts}}) + w^2(D)} \quad (11)$$

Table 5.2 shows typical examples of the expanded relative uncertainty of reference values of four different qualities of force transfer standards in relation to some different types of SI-FSM. The transfer standards with the lowest relative uncertainty achievable to date, as shown in column 2, are force transducers for the range between 2 kN and 2 MN. For the higher force range, build-up systems are available for forces up to 60 MN. If the SI-FSMs are not deadweight machines, the uncertainties of the transfer standards may be less important, as shown in columns 4 and 5. However, in the case of forces above 3 MN, investigations have to be carried out to select the proper transfer standards.

Table 5.1: Examples of expanded relative uncertainty of reference values

	SI-FSM type			
	Deadweight > 2 kN	Deadweight < 2 kN	Lever amplification	Hydraulic amplification
$w(F_{\text{si-fsm}})$	1.0×10^{-5}	1.0×10^{-5}	5.0×10^{-5}	1.0×10^{-4}
$W_{\text{si-fsm}}$	2.0×10^{-5}	2.0×10^{-5}	1.0×10^{-4}	2.0×10^{-4}
$w(X)$	0.3×10^{-5}	0.5×10^{-5}	0.8×10^{-5}	1.7×10^{-5}
W_{ts}	2.1×10^{-5}	2.2×10^{-5}	1.0×10^{-4}	2.0×10^{-4}
a_{drift}	3.0×10^{-5}	5.0×10^{-5}	5.0×10^{-5}	1.0×10^{-4}
$w(D)$	1.2×10^{-5}	2.0×10^{-5}	2.0×10^{-5}	4.1×10^{-5}
W_{rv}	3.2×10^{-5}	4.7×10^{-5}	1.1×10^{-4}	2.2×10^{-4}

After the completion of the calibration of the force calibration machine, its CMC in relative terms may be determined using the following two steps. This calculation is based on the assumption that the force transducer to be calibrated will not interact with the FSM in such a way that it affects the characteristics or measurement uncertainty of the FSM.

Step 4 - Determination of the uncertainty of force generation in the FSM

The output of the calibration of the FSM will be, at each calibrated force, a deviation for incremental forces from the reference value and a deviation for decremental forces from the reference value, both with associated repeatability and reproducibility values. The machine can either be calibrated separately for incremental and decremental forces, in which case the following analysis should be applied only to the direction of interest, or it can be calibrated for both incremental and decremental forces, in which case all calibration results need to be taken into account.

It is highly likely that an FSM will be calibrated using a range of transfer standards of different capacities. When this is the case, there should be common points at which the generated force is measured by two transfer standards. Any difference in the force measured by these two transfer standards is likely to be due to different interaction effects between the transfer standards and the machines and should be carefully assessed prior to incorporation as a separate component in the uncertainty budget.

According to the GUM [8] (note to 6.3.1), corrections should be applied for all known significant systematic effects. If the measurements made in the FSM demonstrate significant deviations between the generated force and the force generated in the SI-FSM, a correction should be made for this deviation – any uncertainty associated with these corrections should be incorporated in the uncertainty budget. It should also be borne in mind that the decremental deviation may not always be known as it could be a function of the maximum force applied during the measurement series. As part of this process, the deviations at forces which were not applied during the machine's calibration, but which are within its range, will need to be estimated to enable correction values to be determined. Depending on the type of machine and any assumptions

relating to the results, different degrees of polynomial fit of deviation against force may be suitable – it is recommended that orders higher than a cubic fit are avoided. The possibilities of fits against partial ranges or in only one direction, for example to compensate for hysteresis, may be considered. The residuals from such fits will enable estimates of uncertainty associated with the calculated corrections to be made. The relative standard uncertainty associated with the correction value at each calibration force is denoted w_{corr} .

If corrections for the measured deviations are not made, and it is strongly recommended that they are made, the deviations cannot simply be treated as uncertainty components because they are known systematic effects. In these cases, a worst-case estimate for the expanded uncertainty at each calibration force can be determined by adding the magnitude of the larger (incremental force (Δd_i) or decremental force (Δd_d)) relative deviation to the expanded uncertainty calculated from all other sources – the absolute value of this magnitude is denoted Δd_{max} . Note that this approach is not that used in F.2.4.5 of the GUM, where a mean deviation across the range is calculated, and the expanded uncertainty incorporates contributions due to the variance of this mean deviation and to the mean variance associated with determining the individual deviation values – this results in an expanded uncertainty associated with the value obtained at each force when using a correction equal to the mean deviation.

The uncertainty contribution due to the lack of reproducibility of the force generated by the FSM is determined from the readings obtained from the transfer standard at a number of rotational positions equally spaced around the machine's measuring axis – this contribution is equal to the standard deviation of the calculated deflections expressed in relative terms and is added to the uncertainty associated with any correction to give the uncertainty associated with the force generation in the FSM (if no correction has been made, w_{corr} is equal to zero):

$$w^2(d_{\text{fsm}}) = \frac{1}{(n-1)} \sum_{i=1}^n \left(\frac{(X_{\text{fsm}_i} - X_{\text{fsm}})}{X_{\text{fsm}}} \right)^2 + w_{\text{corr}}^2 \quad (12)$$

where X_{fsm_i} are the individual deflections obtained at n rotational positions and X_{fsm} is the mean deflection, at each calibration force. It should be noted that the standard deviation value used is that of the sample rather than the mean, as the uncertainty estimation needs to take account of how individual applications of force may vary, rather than the uncertainty associated with their mean value (in contrast to the case in equation (7) with the estimation of the uncertainty associated with the reference value).

Step 5 - Determination of the FSM's CMC

The CMC achieved by deadweight and lever or hydraulic amplification machines is calculated, at each calibrated force, from the following equation (if corrections have been made, Δd_{max} is equal to zero):

$$W_{\text{CMC}} = k \times \sqrt{w_{\text{rv}}^2 + w^2(d_{\text{fsm}})} + \Delta d_{\text{max}} \quad (13)$$

In the calculation for comparator type machines, two additional uncertainty components - the calibration uncertainty $w_{\text{ref_tra}}$ of the reference force transducer and its estimated long-term instability $w_{\text{ref_instab}}$ - must be considered and applied in the following equation:

$$W_{\text{CMC}} = k \times \sqrt{(w_{\text{rv}}^2 + w^2(d_{\text{fsm}}) + w_{\text{ref_tra}}^2 + w_{\text{ref_instab}}^2)} + \Delta d_{\text{max}} \quad (14)$$

Table 5.2 finally shows the typical overall results of the CMC for different types of FSM, assuming that corrections have not been made. The relative uncertainty of the reference force transducer can be calculated using the procedures given in Sections 6 and 7. The long-term instability of the reference force transducer is to be determined from previous calibrations or by estimations.

Table 5.2: Examples of the calibration and measurement capability W_{CMC} for different FSMs

	Deadweight	Lever or hydraulic amplification	Reference transducer system
$W_{\text{ref_tra}}$	—	—	3×10^{-4}
$W_{\text{ref_instab}}$	—	—	2×10^{-4}
W_{rv}	3×10^{-5}	1×10^{-4}	2×10^{-4}
$w(d_{\text{fsm}})$	3×10^{-6}	8×10^{-6}	2×10^{-5}
W_{CMC}	3.1×10^{-5}	1×10^{-4}	4.1×10^{-4}

6 FORCE TRANSDUCERS

This section deals with the uncertainty associated with the results of the calibration of a force transducer in a force calibration machine. Many force transducers are calibrated in accordance with ISO 376, as this is the force traceability route specified in ISO materials testing standards, such as ISO 7500-1 [9] (calibration of uniaxial testing machines) and ISO 6508-2 (calibration of Rockwell hardness testing machines) – Section 6.1 deals with ISO 376 calibrations. There are also other national and international standards covering the calibration of force transducers, such as ASTM E74, BS 8422, and DKD R 3 3 – some brief guidance on the uncertainty estimation approach to be used for these other calibration methods is given in Section 6.2, although much of the technical information given in Section 6.1 will also be applicable to these other methods.

6.1 Determination of the ISO 376 calibration uncertainty

To be consistent with the rest of this document, the guidance given here will be based on a relative uncertainty approach, but it should be borne in mind that a force units approach is equally valid and may be simpler, both for this and for all other force uncertainty estimations in this document.

ISO 376 allows two different analysis methods for the calibration results – one for subsequent use of the transducer only at specific forces and the other enabling it to be used throughout a range of forces, with the applied force being calculated as a specified function of the measured deflection. The definition of the calibration uncertainty is different for these two methods. For instruments classified for interpolation, the calibration uncertainty is the uncertainty associated with the mean increasing force applied in three runs (with the force-proving instrument rotated by 120° between runs

and displaying the same deflection in each run) with the value of this mean force being calculated from the interpolation equation. For instruments classified for specific forces only, the calibration uncertainty is the uncertainty in the value of the mean increasing force applied in three runs (with the force-proving instrument rotated by 120° between runs) when the deflection in each run is equal to one of the mean deflections obtained during the calibration.

At each calibration force, a combined relative standard uncertainty w_c is calculated from the readings obtained during the calibration and other factors, as shown in equation (15).

$$w_c = \sqrt{\sum_{i=1}^8 w_i^2} \quad (15)$$

where:

w_1 = relative standard uncertainty associated with applied calibration force

w_2 = relative standard uncertainty associated with reproducibility of calibration results

w_3 = relative standard uncertainty associated with repeatability of calibration results

w_4 = relative standard uncertainty associated with resolution of indicator

w_5 = relative standard uncertainty associated with creep of instrument

w_6 = relative standard uncertainty associated with drift in zero output

w_7 = relative standard uncertainty associated with temperature of instrument

w_8 = relative standard uncertainty associated with interpolation

Calibration force uncertainty, w_1

w_1 is the relative standard uncertainty associated with the forces applied by the calibration machine. This will generally be equal to the machine's CMC, expressed in relative terms, divided by the value of k specified in the machine's calibration certificate (likely to be equal to 2).

For machines for which the CMC is determined on the basis of corrections not being made (i.e. a non-zero value of Δd_{\max} in equation (13) or (14)), this approach is not strictly correct, but the value determined should still be a reasonable estimate of the calibration force's standard uncertainty.

Reproducibility uncertainty, w_2

w_2 is, at each applied force level, the standard deviation of the mean incremental deflection obtained at equally-spaced orientations in the calibration, expressed as a relative value.

$$w_2 = \frac{1}{|\bar{X}_r|} \times \sqrt{\left(\frac{1}{6} \times \sum_{i=1,3,5} (X_i - \bar{X}_r)^2\right)} \quad (16)$$

where X_i are the deflections obtained in incremental series 1, 3, and 5, and \bar{X}_r is the mean of these three values.

Repeatability uncertainty, w_3

w_3 is, at each applied force level, the contribution due to the repeatability of the measured deflection at a single orientation, expressed as a relative value. It is calculated from:

$$w_3 = \frac{b'}{100 \% \times \sqrt{3}} \quad (17)$$

where b' is the instrument's relative repeatability error, defined in ISO 376 as follows:

$$b' = 100 \% \times \left| \frac{X_2 - X_1}{(X_1 + X_2)/2} \right| \quad (18)$$

where X_1 and X_2 are the deflections obtained at the given force level in series 1 and 2.

Resolution uncertainty, w_4

Each deflection value is calculated as the difference between two readings (the reading at zero force subtracted from the reading at an applied force). The resolution of the indicator therefore needs to be included twice as two rectangular distributions, each with a standard uncertainty of $r/(2\sqrt{3})$ where r is the resolution, expressed in units of force. This is equivalent to one triangular distribution with a standard uncertainty of $r/\sqrt{6}$, and needs to be expressed, at each force level, as a relative value:

$$w_4 = \frac{1}{\sqrt{6}} \times \frac{r}{F} \quad (19)$$

Creep uncertainty, w_5

This uncertainty component is due to the possibility that the instrument's deflection may be influenced by its previous short-term loading history. One measure of this influence is the change in output in the period from 30 s to 300 s after application or removal of the maximum calibration force. This change in output is not included in the reproducibility component because the same calibration machine is generally used for all runs and the time loading procedure will therefore be the same. The magnitude of this uncertainty component can be estimated as follows:

$$w_5 = \frac{c}{100 \% \times \sqrt{3}} \quad (20)$$

where c is the instrument's relative creep error, defined as follows:

$$c = 100 \% \times \left| \frac{i_{300} - i_{30}}{X_N} \right| \quad (21)$$

where i_{30} and i_{300} are the instrument's output 30 s and 300 s respectively after application or removal of the maximum calibration force, and X_N is the deflection at maximum calibration force.

If the creep test is not performed during the calibration, this uncertainty contribution may be estimated as the contribution due to reversibility, given in equation (27), divided by a factor of three.

Zero drift uncertainty, w_6

This uncertainty component is due to the possibility that the instrument's zero output may vary between measurement runs - the subsequent measured deflections may therefore be a function of the time spent at zero force. This effect is not included in the reproducibility component because this time will generally be the same for all runs. One measure of this variation is the ISO 376 zero error f_0 so this effect can be estimated as follows:

$$w_6 = \frac{f_0}{100 \%} \quad (22)$$

where $f_0 = 100 \% \times (i_f - i_o)/X_N$, i_o and i_f are the indicator readings before and after force application respectively, and X_N is the deflection at maximum calibration force.

Temperature uncertainty, w_7

This contribution is due to temperature variation throughout the calibration, together with the uncertainty in the measurement of this calibration temperature range. The sensitivity of the force-measuring instrument to temperature needs to be determined, either by tests or, more commonly, from the manufacturer's specifications. This component takes the same value at each force level and, expressed as a relative value, is equal to:

$$w_7 = K \times \frac{\Delta T}{2} \times \frac{1}{\sqrt{3}} \quad (23)$$

where K is the instrument's temperature coefficient, in K^{-1} , and ΔT is the calibration temperature range, allowing for the uncertainty in the measurement of the temperature. It is worth noting that, for temperature-compensated instruments, this component will generally be negligible (ΔT is unlikely to exceed 2 K and a typical value for K is $0.000\ 05\ K^{-1}$, giving $w_7 = 0.003\ %$, less than the Class 00 calibration force uncertainty contribution).

Interpolation uncertainty, w_8

This uncertainty component is only taken into account for instruments classified for interpolation, as an interpolation equation is not applicable to instruments classified for specific forces only. It is the contribution due to the fitted line not passing exactly through all of the plotted 'applied force' against 'mean deflection' points, and may be calculated using either a residual or deviation method:

Residual method

This method estimates the component using statistical theory. If it is assumed that the calibration forces are evenly distributed, it can be calculated from the following equation:

$$w_8 = \frac{F_N}{F \times X_N} \times \sqrt{\frac{\delta_r}{n - d - 1}} \quad (24)$$

where F_N is the maximum calibration force, F the applied force, X_N is the deflection at maximum calibration force, δ_r is the sum of squared deviations between the mean deflection and the value calculated from interpolation equation, n is the number of force calibration steps, and d is the degree of the equation.

Deviation method

This method estimates the component at each calibration force as the difference between the mean measured deflection, \bar{X}_r , and the value calculated from the interpolation equation, X_a , expressed as a relative value:

$$w_8 = \left| \frac{X_a - \bar{X}_r}{\bar{X}_r} \right| \quad (25)$$

Strictly speaking, the interpolation behaviour of the instrument should not be added in quadrature with the other uncertainty components, as it is a systematic effect at a given calibration force – instead, a correction should be made for this known offset or, failing this, its magnitude should be added to the expanded uncertainty. However, this is the way that it is treated in ISO 376 and, in order not to underestimate its contribution by incorporating it in this manner, the standard uncertainty component is calculated as above, without the division by any coverage factor.

Combined standard uncertainty and expanded uncertainty

At each calibration force, the combined relative standard uncertainty w_c is calculated from equation (15). A graph of w_c against force is plotted and the coefficients of a best-fit least-squares line through all of the data points are determined. The form of the fitted line (i.e. linear, polynomial, exponential) will depend on the calibration results (and it may be easier to achieve a reasonable fit if the combined uncertainty is plotted in force, rather than relative, units). If this results in values that are lower than the minimum value of w_c in any part of the calibration force range, a more conservative fit should be applied or a minimum value for the uncertainty needs to be specified for the relevant parts of the force range.

For any force within the calibration range, the expanded uncertainty W is then calculated from this best-fit line by multiplying its value at the given force by a coverage factor k (usually taken as being equal to 2, assuming that the number of degrees of freedom will be sufficiently large on the basis that, for interpolation classification, this uncertainty fit will be based on at least 32 deflection values).

Table 6.1 gives the relative expanded uncertainty values according to equation (15) for force-proving instruments which only just meet all of the classification criteria given in ISO 376, and so gives the worst-case incremental uncertainty limits for force-proving instruments classified for interpolation. The temperature uncertainty term w_7 is not considered, as its effect on the expanded uncertainty can vary significantly between instruments, from being negligible to being the major contribution.

Table 6.1: Maximum possible expanded uncertainties for instruments classified to ISO 376

Class	w_1	w_2	w_3	w_4	w_5	w_6	w_8	Relative expanded uncertainty
	%	%	%	%	%	%	%	%
00	0.005	0.017	0.014	0.010	0.014	0.012	0.025	0.08
0.5	0.010	0.033	0.029	0.020	0.029	0.025	0.050	0.16
1	0.025	0.067	0.058	0.041	0.058	0.050	0.100	0.32
2	0.050	0.133	0.115	0.082	0.115	0.100	0.200	0.64

6.2 Determination of uncertainty of other calibration procedures

Many other procedures exist for the static or quasi-static calibration of force transducers. However, the method for estimating the uncertainty of the calibration results should be similar to that used in Section 6.1 – the principle which should be borne in mind is that the difference in calibration results from a transducer calibrated to the same procedure in different force calibration machines (within a short period of time) should not be large when compared with the combination of the two calibration uncertainties. In other words, the estimated uncertainties should incorporate all possible differences in the way a transducer can be calibrated but still be within the procedure’s specified criteria – a corollary of this is that, in order to obtain a very low calibration uncertainty, the calibration procedure needs to be very tightly defined. An example of this is the very strictly controlled procedure used in CIPM and RMO key comparisons [10] – this procedure has been specifically developed to minimise the various uncertainty contributions.

Possible uncertainty sources include, but are not limited to, the following:

- Calibration force
- Indicator resolution
- Reproducibility/repeatability of measured deflection
- Creep of transducer
- Effect of zero drift
- Effect of temperature
- How well the interpolation equation fits the data (if applicable)

NOTE: ASTM E74 [11] includes a mandatory method for calculating a lower limit factor (LLF, previously termed “uncertainty”), which it defines as “a statistical estimate of error in forces computed from the calibration equation of a force-measuring instrument when the instrument is calibrated in accordance with these practices.” This calculation of LLF only includes contributions due to reproducibility and deviation from the interpolation equation, although the value is increased to equal the resolution if the original value is calculated to be lower, and the uncertainty of the calibration force applied is also specified to be within certain limits. The method results in a value, in units of force, which is used to determine the lower force limits for the two standard loading ranges (2 000 times the LLF for Class AA and 400 times the LLF for Class A). This method ignores some of the components included in Section 6.1 and, as such,

is likely to result in different, and probably lower, values. The use of only the LLF value associated with the calibration when developing an uncertainty budget for the subsequent use of the force-measuring instrument should be avoided – the contributions due to the other uncertainty components present during the calibration should also be included.

7 INDUSTRIAL FORCE MEASUREMENTS

7.1 Uncertainty contributions to be considered

When the force transducer is used subsequent to its calibration, the uncertainty in the force calculated from its displayed value will depend, in part, on its calibration uncertainty, but there are a number of other factors which also need to be considered. These uncertainty sources include, but are not limited to, the following:

- Resolution
- Contribution due to reversibility
- Drift in sensitivity since calibration
- Effect of being used at a different temperature
- Effect of being used with different end-loading conditions
- Effect of being used with different parasitic components
- Effect of being used with a different time-loading profile
- Effect of linear approximations to interpolation equation
- If applicable, effect of replacement indicator
- Dynamic nature of force being measured

If it can be assumed that none of these effects are correlated, their standard uncertainties can be summed in quadrature, together with the instrument's calibration uncertainty, to calculate a combined standard uncertainty at each force, as given in equation (26). This is based on the assumption that any known errors have been corrected for - for example, if the temperature sensitivity of the transducer is known, and so is the temperature difference (between calibration and subsequent use), either a correction should be made to the calculated force or the magnitude of the effect should be added to the combined expanded uncertainty linearly, rather than being combined in quadrature with the other uncertainty contributions.

$$w_c = \sqrt{\left(\frac{W_{\text{cal}}}{2}\right)^2 + w_{\text{res}}^2 + w_{\text{rev}}^2 + w_{TC_0}^2 + w_{TC_S}^2 + w_{\text{drift}}^2 + \dots} \quad (26)$$

and

$$W = k \times w_c$$

In addition, electromagnetic effects (EMC) can significantly affect the measurement and must be taken into account when developing the uncertainty budget.

A detailed procedure for determining specific uncertainty contributions is given in [12].

Resolution uncertainty w_{res}

The measured force is derived from new deflection values. Because of this, the resolution of the indicator needs to be included again in a similar way to that detailed in 6.1. If the readings fluctuate by more than the resolution of the indicator, the resolution is taken as half the range of fluctuation.

Calculation of contribution due to reversibility w_{rev}

The reversibility error defined in ISO 376 is not treated as a component of the calibration uncertainty. The way to take this contribution into account will depend on how the instrument is used after its calibration.

If the instrument is used to make only increasing measurements, no component due to reversibility needs to be included in the uncertainty of the measured force. However, if measurements of decreasing values of force are made, with no correction based on the calibration results, the uncertainty of the measured force needs to take the reversibility into account by including the following component:

$$w_{\text{rev}} = \frac{v}{100 \% \times \sqrt{3}} \quad (27)$$

where v is the relative reversibility error as defined in ISO 376.

This component is derived purely from the calibration results and may therefore be stated in the instrument's calibration certificate. If required, it could also be added in quadrature to the calibration uncertainty components to obtain an expanded calibration uncertainty which includes the instrument's reversibility although, as with interpolation, it is strictly speaking a systematic effect which would ideally be corrected for.

The reversibility characteristics of a specific force-proving instrument are generally fairly repeatable. Because of this, if the decremental measurements are being made after application of the maximum calibration force (possibly a rare occurrence in industrial force measurements), it may be more effective to make corrections based on the calibration data, rather than to include the whole reversibility effect as an uncertainty contribution. It should also be recognised that the time-loading profile can have a significant effect on the magnitude of the reversibility effect.

Drift in sensitivity since calibration w_{drift}

This contribution can be estimated from the history of the instrument's sensitivity, based on past calibration results. The exact uncertainty distribution (and possibly even an estimated error correction) will depend on the individual instrument, but a rectangular distribution with an expanded uncertainty of \pm the largest previous change between two adjacent calibrations is suggested. If such information is not available, an estimate can be made based on the performance history of similar devices.

Temperature effects w_{TC_0} and w_{TC_S}

The temperature effect on zero output TC_0 can normally be ignored, as the calculation of deflection generally makes it insignificant (except in tests of long duration during which the ambient temperature is changing significantly), but the effect of temperature on sensitivity (or span) TC_S needs to be allowed for. If the actual temperature sensitivity of the instrument is known, a correction should ideally be made to the calculated force. If, as is more likely to be the case, the only information is the manufacturer's specification tolerance, an uncertainty component based on this figure and the

difference in temperature between the instrument's calibration and its subsequent use should be used, with a recommended rectangular distribution. In most cases the TC_0 and TC_S values are given in the datasheet of the sensor as percent per 10 K of full scale and measured value respectively. The uncertainty contributions due to temperature effects are then calculated as follows:

$$w_{TC_0} = \frac{TC_0}{10 \times 100 \%} \times \Delta T_0 \times \frac{F_{nom}}{F_{cal}} \times \frac{1}{\sqrt{3}} \quad (28)$$

$$w_{TC_S} = \frac{TC_S}{10 \times 100 \%} \times \Delta T_{cal} \times \frac{1}{\sqrt{3}} \quad (29)$$

However, the coefficient (or the tolerance) is usually given for a stabilised temperature with no gradient - if the instrument is used in conditions in which it is subject to temperature gradients, an additional uncertainty contribution should be incorporated.

End-loading effect

The bearing pad test specified in ISO 376 gives an indication of the sensitivity of a compression force-proving instrument to specified variations in end-loading conditions. The results of this test, together with information as to the conditions in which the instruments will subsequently be used, should enable realistic uncertainty contributions for use in compression to be estimated. For instruments to be used in tension, it may be necessary to perform additional tests to determine sensitivity to possible variations in force introduction.

Parasitic components effect

The reproducibility component included in the calibration uncertainty is, as explained in 6.1, only valid for a mean of three measurements made on the calibration machine. Larger parasitic components than those applied during calibration are usually applied during the instrument's subsequent use.

It is recommended that the user, where possible, repeat the force measurement, rotating the instrument around the force axis between runs. A component related to any observed variation can then be taken into account.

If it is not possible to repeat measurements with rotation, the magnitude of any parasitic component should be estimated and the sensitivity of the instrument to such parasitic components evaluated or estimated. A component based on the product of the component's magnitude and the instrument's sensitivity should then be included in the uncertainty budget.

Time-loading profile

The force-proving instrument calibration method (as defined in ISO 376) and its subsequent use to verify a uniaxial testing machine (as defined in ISO 7500-1) specify different time-loading profiles (a wait of 30 s before taking a reading in ISO 376, whereas ISO 7500-1 allows calibration with a slowly increasing force). If the load cell is sensitive to time-loading effects, these different methodologies would lead to errors in the calculated force. The creep and zero drift uncertainty contributions in the calibration uncertainty budget will cover these effects, to some degree, but an additional uncertainty contribution may be needed, depending on the particular application.

Care must also be taken if no preload can be applied before the use of the transducer, particularly if it is to be used in both loading modes, i.e. from tension to compression or vice versa.

Effect of approximations to equation

If the calibration equation given in the certificate is not used, a component must be added based on the differences between the calibration equation and the equation that is used in practice.

Some indicators will allow a number of points from the calibration curve to be input, so that the display is in units of force, but will carry out linear interpolation between these points, rather than use the calibration equation. If this is the case, the effect of this linear approximation to the curve should be investigated and, if significant, an uncertainty contribution should be included.

Effect of replacement indicator

If the force transducer is subsequently used with a different indicator than that with which it was calibrated, the deviation between the two indicators must be determined (there are several methods, e.g. calibration of both indicators against electrical standards or use of a common bridge simulator) and the uncertainty of this deviation must be estimated (considering factors such as calibration uncertainty of the indicators or reproducibility of the common bridge simulator).

The uncertainty associated with the value of the generated or measured voltage ratio of this calibration equipment can dominate other uncertainty contributions, significantly increasing the uncertainty of the measured force, particularly in the lower part of the force range. If the resulting uncertainty is too high, the use of a second force transducer to cover this part of the force range could be a solution and should be considered.

It is important that the replacement indicator is of a similar type to the original, with similar settings selected – if this is not the case, additional uncertainty contributions associated with any differences will need to be estimated and incorporated.

If corrections based on the measured deviation between the two indicators are made, the uncertainty of this deviation must be taken into account. If no corrections are made, both the deviation and its uncertainty must be considered.

Calibration uncertainty

This is half the value of the expanded uncertainty calculated in Section 6 using the expanded uncertainty equation.

Effect of dynamic force

If the transducer is used under dynamic conditions, additional contributions have to be taken into account. For example the frequency responses of the force transducer and indicator, and the interaction with the mechanical structure, can strongly influence the measurement results. This requires a detailed analysis of dynamic measurement, which is not covered here.

7.2 Calibration of testing machines to ISO 7500-1

One of the main ISO standards that specifies the use of force-proving instruments calibrated in accordance with ISO 376 is ISO 7500-1 - this details a method to verify the forces generated by uniaxial materials testing machines. Annex C of this standard gives advice on uncertainty estimation, information that is summarised here. An example machine set-up is shown in Figure 6.

ISO 7500-1 permits two ways of calibrating the machine – it is either set to display a nominal value and the transducer is used to measure the generated force ('constant indicated forces'), or the force is increased until the value measured by the transducer is a nominal value and the force displayed by the machine indicator is recorded

(‘constant reference forces’). The first method is recommended and will be discussed here – a similar analysis can be carried out for the second method.

The standard specifies that at least three series of measurements shall be taken with increasing force and, if required, one series shall also be taken with decreasing force. At each force value, the individual accuracy errors and the repeatability error are calculated, as is, if required, the reversibility error – together with the proving instrument classification, the zero error, and the machine resolution, these can be used to determine the machine’s classification.

The uncertainty associated with the machine calibration for incremental forces, as suggested in Annex C, is the uncertainty associated with the estimate of the relative accuracy error at each calibration force. This is based on, as a minimum, the repeatability of the results, the resolution of the machine indicator, and the contributions due to the transfer standard – these transfer standard contributions include its calibration uncertainty, its sensitivity to temperature, any sensitivity drift since its calibration, and any effects due to approximations to the interpolation equation. These contributions are all covered in Section 7.1 – the other items in that section should also be considered when estimating an uncertainty value for the machine calibration.

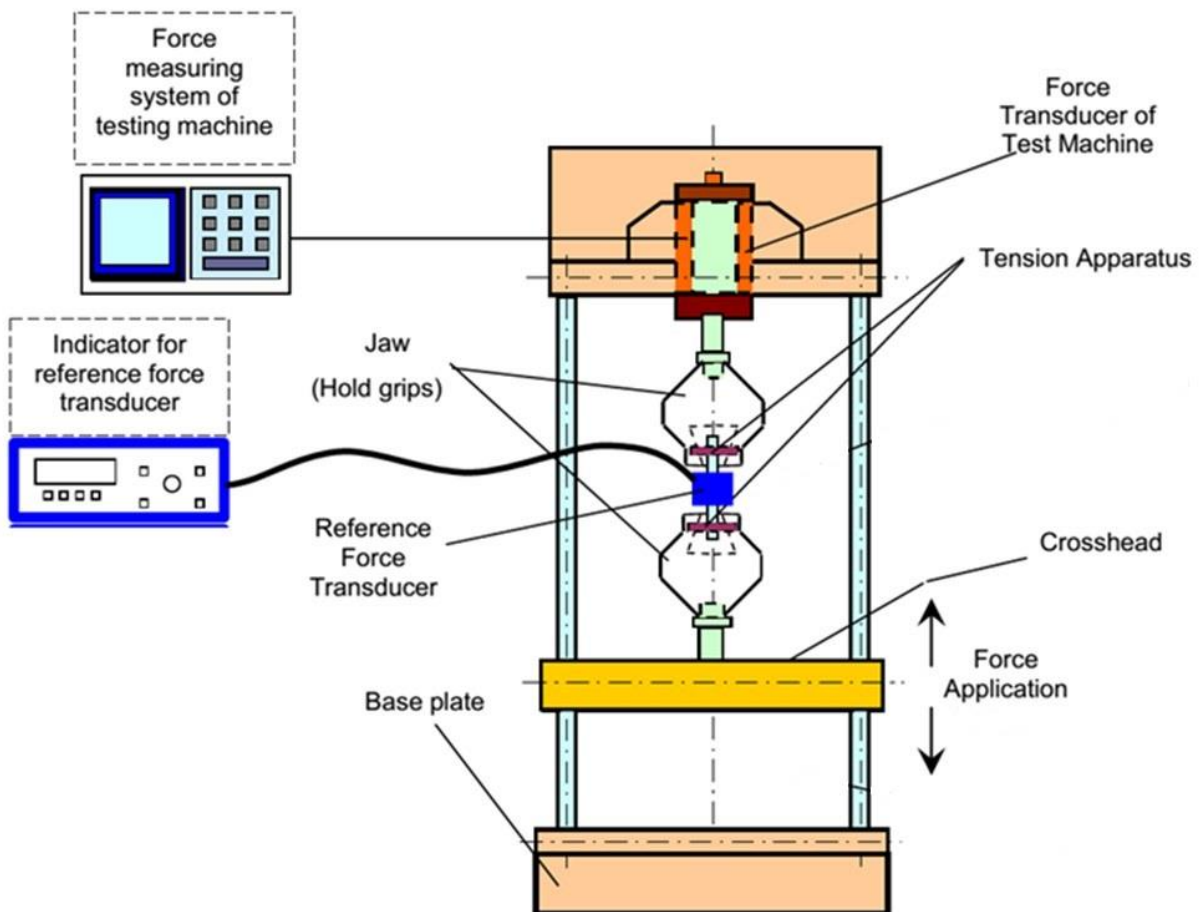


Figure 6. Material testing machine - tension calibration set up

Annex C calculates the calibration uncertainty as follows:

$$W = k \times w_c = k \times \sqrt{(w_{\text{rep}}^2 + w_{\text{res}}^2 + w_{\text{std}}^2)} \quad (30)$$

where:

- w_{rep} is the standard deviation of the mean error at a given force, expressed as a relative value
- w_{res} is the contribution due to resolution, given by:

$$w_{\text{res}} = \sqrt{\left(\left(a_F/\sqrt{12}\right)^2 + \left(a_z/\sqrt{12}\right)^2\right)} \quad (31)$$

where:

- a_F is the relative resolution of the machine's indicator at the applied force
- a_z is the relative resolution (as a proportion of the calibration force) of the machine's indicator at zero force
- w_{std} is the contribution due to the transfer standard, given by:

$$w_{\text{std}} = \sqrt{(w_{\text{cal}}^2 + w_{\text{temp}}^2 + w_{\text{drift}}^2 + w_{\text{approx}}^2)} \quad (32)$$

where:

- w_{cal} is the transfer standard's calibration uncertainty
- w_{temp} is the uncertainty due to temperature effects
- w_{drift} is the uncertainty due to drift of the standard's sensitivity
- w_{approx} is the effect of approximating to the interpolation equation

7.3 Other industrial force measurement applications

In other industrial force measurement applications, similar uncertainty contributions will need to be considered. The basic philosophy is that the transducer will introduce a specific uncertainty based on its calibration results, and then there will be further uncertainty contributions due to the transducer being used at a different time and under different conditions to those experienced during its calibration – the magnitudes of these various contributions need to be estimated and, if it can be demonstrated that they are not correlated, then combined in quadrature to obtain a combined standard uncertainty for the measurement result. This standard uncertainty can then be multiplied by a coverage factor to give an expanded uncertainty at the required confidence level.

One of the major differences in conditions between calibration and use may be that the transducer has been calibrated under a fairly static force regime (probably due to the unavailability of dynamic standard facilities and/or calibration methods) but is used to make measurements of rapidly-changing, or dynamic, forces. Examples of such applications include the force measurement system in dynamic testing machines (such as fatigue machines), industrial presses, and road load data collection equipment. The uncertainty associated with the force measurement value will need to include components relating to such dynamic effects, but this is best done on a case-by-case

basis – this major area of uncertainty analysis cannot be covered in full here, and readers are encouraged to consult the relevant references for further information.

8 REFERENCES AND FURTHER READING

8.1 References

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- [9] ISO 7500-1:2018. *Metallic materials - Calibration and verification of static uniaxial testing machines – Part 1: Tension/compression testing machines – Calibration and verification of the force-measuring system.*
- [10] BIPM Key Comparison Database (KCDB): <https://www.bipm.org/kcdb/comparison/quick-search>
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- [12] Röske D, Ala-Hiiri J, Knott A, Medina N, Kaspar P and Wozniak M: *Tools for uncertainty calculations in force measurement*, ACTA IMEKO Vol 6, No 1 (2017): <https://acta.imeko.org/index.php/acta-imeko/article/view/IMEKO-ACTA-06%20%282017%29-01-09>

8.2 Further reading

Sawla, A.: *Guidance for the determination of the best measurement capability of force calibration machines and uncertainty of calibration results of force measuring devices*. Braunschweig, PTB-Mitteilungen 104 4/94, 1994.

Sawla, A.: *Uncertainty of measurement in the verification and calibration of the force-measuring systems of testing machines*. Proc. of the Asia-Pacific Symposium on Measurement of Force, Mass, and Torque (APMF 2000), pp 7-14. Tsukuba, Japan, November 2000.

Annex A – Example ISO 376 uncertainty calculations

This annex details the uncertainty calculations associated with an example single mode (i.e. tension or compression) force-proving instrument calibration in accordance with ISO 376. The calibration is performed in a deadweight force standard machine with a certified expanded uncertainty of applied force of 0.002 % at a stable measured temperature of 20.0 °C.

The following table gives the deflections obtained during the calibration – these have been calculated by subtracting the initial output at zero force from each subsequent output obtained during the calibration run.

Force	Series 1	Series 2	Series 3	Series 4	Series 5	Series 6
	0°	0°	120°	120°	240°	240°
	Inc.	Inc.	Inc.	Dec.	Inc.	Dec.
kN	mV/V	mV/V	mV/V	mV/V	mV/V	mV/V
0	0.000 00	0.000 00	0.000 00	0.000 04	0.000 00	0.000 04
2	0.200 09	0.200 13	0.200 16	0.200 20	0.200 10	0.200 17
4	0.400 28	0.400 27	0.400 35	0.400 39	0.400 29	0.400 33
6	0.600 49	0.600 46	0.600 54	0.600 63	0.600 47	0.600 62
8	0.800 69	0.800 67	0.800 74	0.800 84	0.800 72	0.800 86
10	1.000 95	1.000 92	1.000 94	1.001 07	1.000 94	1.001 09
12	1.201 15	1.201 14	1.201 15	1.201 32	1.201 17	1.201 25
14	1.401 35	1.401 36	1.401 37	1.401 56	1.401 39	1.401 47
16	1.601 55	1.601 55	1.601 57	1.601 74	1.601 62	1.601 68
18	1.801 79	1.801 77	1.801 76	1.801 89	1.801 80	1.801 86
20	2.001 99	2.001 97	2.001 99	n/a	2.002 05	n/a
0	0.000 07	0.000 08	n/a	n/a	n/a	n/a

The following table gives calculated values for the mean deflection with and without rotation (\bar{X}_r and \bar{X}_{wr} respectively), together with the interpolation values (X_a) derived from the quadratic fit (in which F is in kN and X_a is in mV/V):

$$X_a = 0.000\ 000\ 19 \times F^2 + 0.100\ 101\ 7 \times F - 0.000\ 1$$

Force	\bar{X}_r	\bar{X}_{wr}	X_a
kN	mV/V	mV/V	mV/V
2	0.200 12	0.200 11	0.200 10
4	0.400 31	0.400 28	0.400 31
6	0.600 50	0.600 48	0.600 52
8	0.800 72	0.800 68	0.800 73
10	1.000 94	1.000 94	1.000 94
12	1.201 16	1.201 15	1.201 15
14	1.401 37	1.401 36	1.401 36
16	1.601 58	1.601 55	1.601 58
18	1.801 78	1.801 78	1.801 79
20	2.002 01	2.001 98	2.002 01

A creep test was carried out prior to the calibration, with the output values on unloading from maximum force being, after 30 s and 300 s, 0.019 42 mV/V and 0.019 30 mV/V respectively.

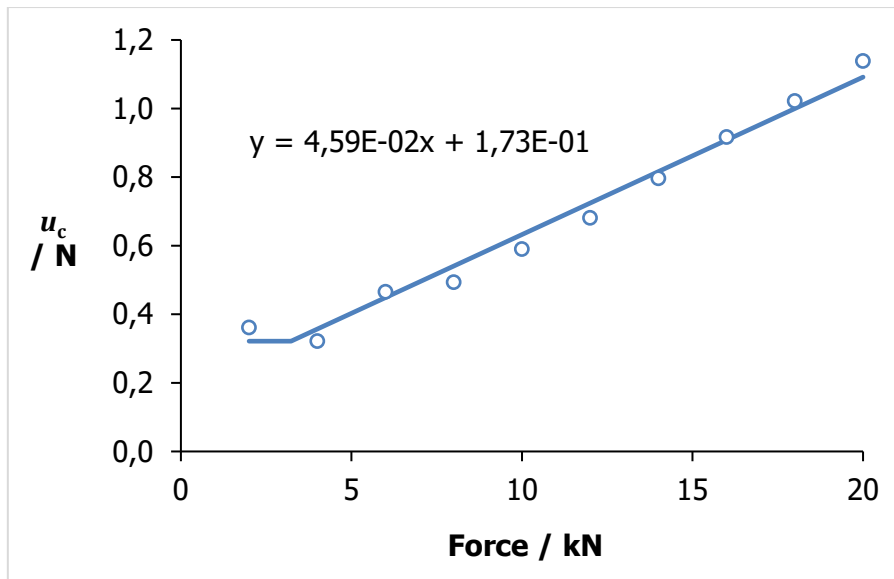
The manufacturer's specification for the transducer is that its temperature sensitivity does not exceed 0.01 %·K⁻¹, and the calibration temperature range, allowing for the uncertainty in the temperature measurement, is estimated to be 0.5 K.

From this information, the uncertainty contributions at each calibration force can be calculated as described in the following table.

Component	Symbol	Calculation of component in percentage terms
Calibration force	w_1	expanded uncertainty of applied force / 2 = 0.001 %
Reproducibility	w_2	standard deviation of mean incremental deflection, expressed as a relative value
Repeatability	w_3	repeatability at 0° divided by $\sqrt{3}$
Resolution	w_4	resolution divided by $\sqrt{6}$, expressed as a relative value
Creep	w_5	relative creep error divided by $\sqrt{3}$
Zero drift	w_6	ISO 376 zero error
Temperature	w_7	temperature sensitivity × temperature range divided by $2\sqrt{3}$
Interpolation	w_8	interpolation error, expressed as a relative value

The individual relative uncertainty contributions can then be used to estimate a combined relative uncertainty w_c at each calibration force, as shown in the following table. The final column expresses the combined uncertainty in force units (u_c) and the subsequent graph plots these values against the calibration force and displays the equation of a linear fit to this data, a fit which is truncated to be equal to the minimum combined uncertainty value (0.32 N at the 4 kN calibration point). To avoid the need for this truncation, a quadratic fit to the data could alternatively be employed.

Force	w_1	w_2	w_3	w_4	w_5	w_6	w_7	w_8	w_c	u_c
kN	%	%	%	%	%	%	%	%	%	N
2	0.001	0.011	0.012	0.002	0.003	0.004	0.001	0.006	0.018	0.36
4	0.001	0.005	0.001	0.001	0.003	0.004	0.001	0.001	0.008	0.32
6	0.001	0.003	0.003	0.001	0.003	0.004	0.001	0.003	0.008	0.47
8	0.001	0.002	0.001	0.001	0.003	0.004	0.001	0.001	0.006	0.49
10	0.001	0.000	0.002	0.000	0.003	0.004	0.001	0.001	0.006	0.59
12	0.001	0.001	0.000	0.000	0.003	0.004	0.001	0.001	0.006	0.68
14	0.001	0.001	0.000	0.000	0.003	0.004	0.001	0.001	0.006	0.80
16	0.001	0.001	0.000	0.000	0.003	0.004	0.001	0.000	0.006	0.92
18	0.001	0.001	0.001	0.000	0.003	0.004	0.001	0.001	0.006	1.02
20	0.001	0.001	0.001	0.000	0.003	0.004	0.001	0.000	0.006	1.14



To determine the expanded uncertainty associated with the calibration results, the coefficients of this linear fit are doubled ($k = 2$), enabling U to be calculated at each calibration force. These values can also be converted into relative ones, as shown in the following table.

Force kN	w_c %	u_c N	u_c (fit) N	U N	W %
2	0.018	0.36	0.32	0.6	0.032
4	0.008	0.32	0.36	0.7	0.018
6	0.008	0.47	0.45	0.9	0.015
8	0.006	0.49	0.54	1.1	0.014
10	0.006	0.59	0.63	1.3	0.013
12	0.006	0.68	0.72	1.4	0.012
14	0.006	0.80	0.82	1.6	0.012
16	0.006	0.92	0.91	1.8	0.011
18	0.006	1.02	1.00	2.0	0.011
20	0.006	1.14	1.09	2.2	0.011

Within the calibrated force range (2 kN to 20 kN), the expanded uncertainty associated with the value of force derived from the interpolation equation can be calculated at any force, in either force (U) or relative (W) terms, using the following equations:

$2.0 \text{ kN} \leq F < 3.2 \text{ kN}$	$3.2 \text{ kN} \leq F \leq 20 \text{ kN}$
$U = 0.6 \text{ N}$	$U = (0.092 F/\text{kN} + 0.35) \text{ N}$
$W = (0.064/(F/\text{kN})) \%$	$W = (0.0092 + 0.035/(F/\text{kN})) \%$

Annex B – Example ISO 7500-1 uncertainty calculations

The force-proving instrument calibrated in Annex A (but with its calibration uncertainty increased by a factor of ten, to give more realistic results) is now used to verify the forces generated by a testing machine at a stable measured temperature of 19.5 °C, with the following table giving, at each nominal force in each of three incremental series, the machine's displayed force and the instrument's measured output:

	Series 1		Series 2		Series 3	
Nominal Force	Displayed Force	Output	Displayed Force	Output	Displayed Force	Output
kN	kN	mV/V	kN	mV/V	kN	mV/V
0	0.00	0.000 00	0.00	0.000 00	0.00	0.000 00
2	2.00	0.199 24	2.00	0.199 28	2.00	0.199 12
3	3.00	0.297 93	3.01	0.299 52	3.01	0.300 23
4	4.01	0.397 77	4.01	0.398 76	4.01	0.399 41
5	4.99	0.495 38	5.00	0.498 30	5.01	0.499 97
6	6.00	0.596 42	6.01	0.599 03	6.00	0.599 26
7	7.01	0.697 51	7.01	0.697 98	7.01	0.697 77
8	8.01	0.796 21	8.01	0.798 27	8.00	0.797 36
9	9.01	0.896 55	9.01	0.896 45	9.01	0.897 80
10	10.01	0.996 01	10.01	0.997 85	10.00	0.995 29
0	0.00	-0.000 39	0.03	0.000 12	0.01	-0.000 10

The instrument's outputs are then converted to force values using its calibration coefficients and all individual errors are calculated, together with the mean error and its standard deviation (which is then used as the repeatability component in the uncertainty estimation) at each nominal force (the various symbols used for the error terms are those employed in ISO 7500-1):

	Series 1		Series 2		Series 3		Mean	
Nominal Force	Generated Force	Error q_1	Generated Force	Error q_2	Generated Force	Error q_3	Error q	St. Dev.
kN	kN	%	kN	%	kN	%	%	%
2	1.991	0.43	1.992	0.41	1.990	0.49	0.44	0.04
3	2.977	0.76	2.993	0.56	3.000	0.33	0.55	0.22
4	3.975	0.88	3.985	0.64	3.991	0.47	0.67	0.21
5	4.950	0.81	4.979	0.42	4.996	0.29	0.51	0.27
6	5.959	0.68	5.985	0.41	5.987	0.21	0.44	0.24
7	6.969	0.59	6.974	0.52	6.972	0.55	0.55	0.03
8	7.955	0.69	7.975	0.43	7.966	0.42	0.51	0.15
9	8.957	0.59	8.956	0.60	8.970	0.45	0.54	0.08
10	9.951	0.59	9.969	0.41	9.944	0.56	0.52	0.10

The following tables detail the uncertainty components used in the uncertainty estimation and the calculation of the expanded uncertainty W associated with the mean error value:

Component	Symbol	Calculation of component in percentage terms
Repeatability	w_{rep}	standard deviation of mean error, expressed as a relative value
Resolution	w_{res}	relative resolutions at zero and applied force, both divided by $\sqrt{12}$ then added in quadrature
Calibration	w_{cal}	transfer standard's relative calibration uncertainty
Temperature	w_{temp}	difference from transfer standard's calibration temperature multiplied by temperature sensitivity and divided by $\sqrt{3}$
Drift	w_{drift}	largest transfer standard sensitivity change between previous calibrations (0.1 % in this example) divided by $\sqrt{3}$
Linear Approximation	w_{approx}	component to compensate for transfer standard's exact calibration equation not being used at all points (not always needed)

Nominal Force	w_{rep}	w_{res}	w_{cal}	w_{temp}	w_{drift}	w_{approx}	w_c	W
kN	%	%	%	%	%	%	%	%
2	0.024	0.204	0.160	0.003	0.058	0.000	0.267	0.534
3	0.125	0.136	0.107	0.003	0.058	0.000	0.221	0.442
4	0.119	0.102	0.089	0.003	0.058	0.000	0.189	0.379
5	0.155	0.082	0.081	0.003	0.058	0.000	0.201	0.403
6	0.137	0.068	0.075	0.003	0.058	0.000	0.180	0.360
7	0.019	0.058	0.071	0.003	0.058	0.000	0.110	0.220
8	0.088	0.051	0.068	0.003	0.058	0.000	0.135	0.270
9	0.048	0.045	0.065	0.003	0.058	0.000	0.109	0.219
10	0.057	0.041	0.063	0.003	0.058	0.000	0.111	0.222

The following table summarises the final calibration results:

Nominal Force	Mean Error			
	Value <i>q</i> %	Expanded Uncertainty <i>W</i> %	Value N	Expanded Uncertainty N
kN				
2	0.44	0.53	9	11
3	0.55	0.44	16	13
4	0.67	0.38	27	15
5	0.51	0.40	25	20
6	0.44	0.36	26	22
7	0.55	0.22	39	15
8	0.51	0.27	41	22
9	0.54	0.22	49	20
10	0.52	0.22	52	22

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